

HYBRID QUINTIC B-SPLINE APPROACH FOR THE NUMERICAL SOLUTION OF KURAMOTO-SIVASHINSKY EQUATION

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Abstract

This article focuses on finding an approximate solution to the Kuramoto–Sivashinsky (KS) equation. Smoothly connected polynomial segments called B-spline functions are used and controlled by a set of points. The KS equation is a nonlinear fourth-order partial differential equation which has been used extensively to model various phenomena including flame-front instability, thin film flow and phase dynamics. The hybrid basis consists of a convex combination of polynomial quintic B-splines and trigonometric quintic B-splines. The numerical approximation of the time is done by the Crank–Nicolson finite difference method and the linearization of the nonlinear term is realized by the Rubin–Graves technique. The approximation of spatial derivatives is done with HQBS functions. Von Neumann approach is widely used to study the stability of a numerical method. The accuracy and effectiveness of the scheme is confirmed by the numerical results, which are accurate and close to the analytical solutions.

Keywords:

Kuramoto–Sivashinsky equation, hybrid quintic B-spline, trigonometric B-spline, collocation method, Crank–Nicolson scheme, Rubin–Graves linearization, stability analysis.

1. INTRODUCTION

Nonlinear partial differential equations (PDEs) and their numerical solutions like the Kuramoto-Sivashinsky equation has been a subject of intense research in recent years, especially when using quintic B-spline methods. These techniques are known for their efficiency and accuracy in dealing with complex nonlinear dynamics, and thus are applicable to a range of areas of mathematical physics and engineering.

To solve the Kuramoto–Sivashinsky equation Lakestani and Dehghan used b-spline functions, and the results of the simulation were a good contract with the exact solution [1]. In [2], Hepson solved some initial boundary value problems for Kuramoto-sivashinsky equation to find the numerical solution by used the B-spline algorithm.

Mittal and Dahiya [3] found approximate results of the Kuramoto–Sivashinsky equation applying by a quadratic technique with basic functions being quintic B-splines. Akrivis and Smyrlis in [4] Used implicit–explicit backward differentiation formula (IMEX-BDF) schemes to get accurate arithmetical solutions for the KS equation. In [5], Khater and Temsah used Chebyshev spectral collocation approach to numerically solve the Kuramoto sivashinsky equation. Khater and Temsah [6] using modified 4th order B-spline collocation method calculated numerical solution Kuramoto-Sivashinsky equation. Hepson [7] used Galerkin finite element method to obtain a numerical solution to Kuramoto-sivashinsky. Mahmood *et al.* [8] Suggested a new hybrid numerical technique for approximate answer of KS- equation using non-polynomial splines finite difference methods. M. zarabnia [9] Numerically solved the KS- equation using a collocation method with 4th B-spline functions. Marcos [10] investigated pseudospectral spatial discretization methods for the Kuramoto–Sivashinsky equation.

Choudhary and Kumar presented a numerical method to resolve the time fractional linear KSE. The first time fractional derivative of order γ is applied Caputo's logic in [11]. Komal *et al.* introduced a modified ABC-operator to substitute the time-fractional derivative and used a numerical technique to solve the time-fractional KSE [12]. In [13], Kaur and Kumar have applied a hybrid numerical scheme to solve Kuramoto–Sivashinsky equation. Ishwarya *et al.* [14] modified the quintic B-spline differential quadrature method to find numerical solutions for the nonlinear generalized Kuramoto–Sivashinsky equation. Jena and Gebremedhin in [15] Approximate answer of the KSE was obtained by the B-spline collocation method. This is achieved by linearising the nonlinear part of KSE using Taylor series method. In [16], Abed used hybrid PSO-NIM method to find the optimum parameters for the non-linear Kuramoto Sivashinsky equation. Bhatt and Chowdhury develop a numerical scheme to analyze the spatio-temporal behavior of the Kuramoto Sivashinsky equation in [17]. Islam *et al.* applied the mesh-free radial basis function (RBF) method to the Kuramoto–Sivashinsky equation [18]. Pani *et al.* applied a 2nd order splitting method combined with cubic- spline collocation to solved the Kuramoto Sivashinsky equation in [19]. Kumar *et al.* developed a sophisticated numerical method for the time-fractional KSE that used the Caputo fractional derivative [20]. Xu and shu used local discontinuous Galerkin method to solve the Kuramoto–Sivashinsky and Ito-type coupled KdV equations in [21].

The kuramoto-sivashinsky (KS) equation is a nonlinear fourth-order parabolic PD equation. Its general form is:

$$v_t + vv_z + \rho v_{zz} + uv_{zzzz} = 0, \quad (1)$$

with

$$z \in [a, b], \quad t \in [0, T] \quad (2)$$

where $v=v(x,t)$ is the dependent variable (an interface height, flame-front perturbation, or phase variable, depending on the physical context), and ρ, u, v constant coefficients

With the BC

$$v(z_0, t) = g_0 \quad v(z_N, t) = g_1 \quad (3)$$

$$v_z(z_0, t) = 0 \quad v_z(z_N, t) = 0 \quad (4)$$

$$v_{zz}(z_0, t) = 0 \quad v_{zz}(z_N, t) = 0 \quad (5)$$

And the initial condition

$$v(z, 0) = u_o, \quad z \in [a, b] \quad (6)$$

2. DESCRIPTION OF METHOD

The solution domain $a \leq x \leq b$ is partitioned in to a mesh of uniform length $h = x_{i+1} - x_i$, by the knots x_i where $i = 0, 1, 2, \dots, N$ such that $a = z_0 < z_1 \cdots z_{n-1} < z_N = b$.

We aim to determine an approximate solution $V_m(z, t)$ for the KS equation by employing the method with a hybrid 4th order B-spline in our numerical approach to the exact answer $v(z, t)$ in the form:

$$V_m(z, t) = \sum_{j=-2}^{N+2} \omega_j(t) H_j(z) \quad (7)$$

The end conditions of the differential equations are used to determine the time-dependent quantities $\omega_i(t)$

And $H_j(x)$ are the hybrid quintic B-spline base functions at given as

$$H_j(z) = \sigma D_j(z) + (1 - \sigma) T_j(z) \quad (8)$$

Where the $T_i(z)$ is the trigonometric quintic B-spline and $D_i(z)$ are the quintic B-spline function.

The values of σ plays important rules when the value of $\sigma = 0$ the function will be TQBS and if the value of $\sigma = 1$ the function will be QBS but here take the value $0 < \sigma < 1$.

$D_i(x)$ Are the 4th order Bspline functions loops are assumed by

$$D_j(z) = \frac{1}{120h^5} \begin{cases} (z - z_{j-3})^5 & z \in [z_{j-3}, z_{j-2}), \\ (z - z_{j-3})^5 - 6(z - z_{j-2})^5 & z \in [z_{j-2}, z_{j-1}), \\ (z - z_{j-3})^5 - 6(z - z_{j-2})^5 + 15(z - z_{j-1})^5 & z \in [z_{j-1}, z_j), \\ (z_{j+3} - z)^5 - 6(z_{j+2} - z)^5 + 15(z_{j+1} - z)^5 & z \in [z_j, z_{j+1}), \\ (z_{j+3} - z)^5 - 6(z_{j+2} - z)^5 & z \in [z_{j+1}, z_{j+2}), \\ (z_{j+3} - z)^5 & z \in [z_{j+2}, z_{j+3}), \\ 0 & \text{otherwise.} \end{cases} \quad (9)$$

And $T_i(z)$ are the trigonometric 4th order B-spline function at loops are assumed by

$$T_j(z) = \frac{1}{Q} \begin{cases} X^5(z_{-3+j}), & z \in [z_{-3+j}, z_{-2+j}), \\ X^4(z_{-3+j})\zeta(z_{-1+j}) + X^3(z_{-3+j})\zeta(z_j)X(z_{-2+j}) + X^2(z_{-3+j})\zeta(z_{1+j})X^2(z_{-2+j}) \\ + X(z_{-3+j})\zeta(z_{2+j})X^3(z_{-2+j}) + \zeta(z_{3+j})X^4(z_{-2+j}), & z \in [z_{-2+j}, z_{-1+j}), \\ X^3(z_{-3+j})\zeta^2(z_j) + X^2(z_{-3+j})\zeta(z_{1+j})X(z_{-2+j})\zeta(z_j) \\ + X^2(z_{-3+j})\zeta^2(z_{1+j})X(z_{-1+j}) + X(z_{-3+j})\zeta(z_{2+j})X^2(z_{-2+j})\zeta(z_j) \\ + X(z_{-3+j})\zeta(z_{2+j})X(z_{-2+j})\zeta(z_{1+j})X(z_{-1+j}) + X(z_{-3+j})\zeta^2(z_{2+j})X^2(z_{-1+j}) \\ + \zeta(z_{3+j})X^3(z_{-2+j})\zeta(z_j) + \zeta(z_{3+j})X^2(z_{-2+j})\zeta(z_{1+j})X(z_{-1+j}) \\ + \zeta(z_{3+j})X(z_{-2+j})\zeta(z_{2+j})X^2(z_{-1+j}) + \zeta^2(z_{-3+j})X^3(z_{-3+j}), & z \in [z_j, z_{1+j}), \\ X(z_{-3+j})\zeta^4(z_{2+j}) + \zeta(z_{3+j})X(z_{-2+j})\zeta^3(z_{2+j}) + \zeta^2(z_{3+j})X(z_{-1+j})\zeta^2(z_{2+j}) \\ + \zeta^3(z_{3+j})X(z_j)\zeta(z_{2+j}) + \zeta^4(z_{3+j})X(z_{1+j}), & z \in [z_{1+j}, z_{2+j}), \\ \zeta^5(z_{3+j}), & z \in [z_{2+j}, z_{3+j}), \\ 0 & \text{otherwise,} \end{cases} \quad (10)$$

Where $Q = \sin(2h/4)\sin(h)\sin(2h)\sin(6h/4)\sin(10h/4)$, $X(z_j) = \sin(\frac{z - z_i}{2})$ and $\zeta(z_i) = \sin(\frac{z_j - z}{2})$

obviously all $T_j(z)$ is piecewise TQB-spline and $T_j(z) \in X$, a linear subspace of $L_2[-1, 1]$.

Table 1:

When the Values of the polynomial building block $D_i(x)$ and its derivatives at the knots

Setting $\sigma = 1$ in $H_j(z) = \sigma D_j(z) + (1 - \sigma)T_j(z)$ isolates the ordinary polynomial quintic B-spline $D_j(z)$

Nodel values of the QBS and its derivatives.

Table.1

	z_{j-3}	z_{j-2}	z_{j-1}	z_j	z_{j+1}	z_{j+2}	z_{j+3}
$120D_j$	0	1	26	66	26	1	0
$120hD'_j$	0	5	50	0	-50	-5	0
$120h^2D''_j$	0	20	40	-120	40	20	0
$120h^3D'''_j$	0	60	-120	0	120	-60	0
$120h^4D^{iv}_j$	0	120	-480	720	-480	120	0

The trigonometric quintic B-spline derivaties are

Table.2

	z_{-3+i}	z_{-2+i}	z_{-1+i}	z_i	z_{1+i}	z_{2+i}	z_{3+i}
T_j	0	β_1	β_2	β_3	β_2	β_1	0
T'_j	0	$-\beta_4$	$-\beta_5$	0	β_5	β_4	0
T''_j	0	β_6	β_7	β_8	β_7	β_6	0
T'''_j	0	$-\beta_9$	β_{10}	0	$-\beta_{10}$	β_9	0
T^{iv}_j	0	β_{11}	β_{12}	β_{13}	β_{12}	β_{11}	0

The coefficients $\beta_1, \dots, \beta_{13}$

Writing $c = \cos \frac{h}{2}$, $s = \sin \frac{h}{2}$, and using the normalizer Q defined above, the thirteen coefficients are

$$\beta_1 = \frac{s^5}{Q} \quad \beta_2 = \frac{2s^5c(16c^2-3)}{Q} \quad \beta_3 = \frac{2(1+48c^4-16c^2s^5)}{Q}$$

$$\begin{aligned}\beta_4 &= \frac{5/2 s^4 c}{Q} & \beta_5 &= \frac{5s^4 c^2(8c^2 - 3)}{Q} & \beta_6 &= \frac{5/4 s^3(5c^2 - 1)}{Q} \\ \beta_7 &= \frac{5/2 s^3[c(-15c^2 + 3 + 16c^4)]}{Q} & \beta_8 &= \frac{-5/2 s^3(16c^2 - 5c^6 + 1)}{Q} \\ \beta_9 &= \frac{5/8 s^2 c(25c^2 - 13)}{Q} & \beta_{10} &= \frac{-5/4 s^2[c^2(8c^4 - 35c^2 + 15)]}{Q} \\ \beta_{11} &= \frac{5/16 s(125c^4 - 144c^2 + 13)]}{Q} & \beta_{12} &= \frac{-5/8 s c(176c^6 - 137c^4 - 6c^2 + 15)}{Q} \\ \beta_{13} &= \frac{5/8(192c^6 - 117c^4 + 62c^2 - 13)(-1 + 4c^2 s)}{Q}\end{aligned}$$

By approximate purpose (7) and hybrid quintic b-spline (8) the estimated values at nubs $V(x)$ of and its derivative upto 4th order are determined in terms of the time parameters ω_m as

$$V_m = \beta_1 \omega_{-2+m} + \beta_2 \omega_{-1+m} + \beta_3 \omega_m + \beta_2 \omega_{1+m} + \beta_1 \omega_{2+m} \quad (11)$$

$$V'_m = -\beta_4 \omega_{-2+m} - \beta_5 \omega_{-1+m} + \beta_5 \omega_{1+m} + \beta_4 \omega_{2+m} \quad (12)$$

$$V''_m = \beta_6 \omega_{-2+m} + \beta_7 \omega_{-1+m} + \beta_8 \omega_m + \beta_7 \omega_{1+m} + \beta_6 \omega_{2+m} \quad (13)$$

$$V'''_m = -\beta_9 \omega_{-2+m} + \beta_{10} \omega_{-1+m} - \beta_{10} \omega_{1+m} + \beta_9 \omega_{2+m} \quad (14)$$

$$V^{iv}_m = \beta_{11} \omega_{-2+m} + \beta_{12} \omega_{-1+m} + \beta_{13} \omega_m + \beta_{12} \omega_{1+m} + \beta_{11} \omega_{2+m} \quad (15)$$

4. CRANK–NICOLSON TIME DISCRETIZATION AND LINEARIZATION

Using the recommended approach, discretize derivative with respect to time using the standard FD-method and use the CN-scheme to (1), we develop

$$\left[\frac{v^{1+n} - v^n}{\Delta t} \right] + \left[\frac{(v v_z)^{1+n} + (v v_z)^n}{2} \right] + \rho \left[\frac{(v_{zz})^{1+n} + (v_{zz})^n}{2} \right] + u \left[\frac{(v_{zzzz})^{1+n} + (v_{zzzz})^n}{2} \right] = 0, \quad (16)$$

where Δt the interval of time

linearize non-linear term $(v v_z)^{1+n}$ we employ Rubin and Graves' linearization form (16)

$$(v v_z)^{n+1} = v^{n+1} v_z^n + v^n v_z^{n+1} - (v v_z)^n. \quad (17)$$

Placing values Equation (17) in (16) it becomes

$$\left[\frac{v^{n+1} - v^n}{\Delta t} \right] + \left[\frac{v^{n+1}v_z + v^n v_z^{n+1}}{2} \right] + \rho \left[\frac{(v_{zz})^{n+1} + (v_{zz})^n}{2} \right] + u \left[\frac{(v_{zzzz})^{n+1} + (v_{zzzz})^n}{2} \right] = 0. \quad (18)$$

Readjusting the terms and shortening we get,

$$v^{n+1} + \frac{\Delta t}{2} \left[v^{n+1}v_z^n + v^n v_z^{n+1} + \rho(v_{zz})^{n+1} + u(v_{zzzz})^{n+1} \right] = v^n - \frac{\Delta t}{2} \left[\rho(v_{zz})^n + u(v_{zzzz})^n \right]. \quad (19)$$

By replacing v with the approximate solution V their derivatives and entering the nodal values V

5. THE PENTA-DIAGONAL SYSTEM IN THE HYBRID BASIS

By replacing v with the approximate solution V their derivatives and entering the nodal values V using Eqs. (11)–(15) Freeze the local slope and value at the previous time level,

$$p = (V'_m)^n \quad q = V_m^n$$

And we defined

$$A = 1 + \frac{\Delta t}{2} p \quad B = \frac{\Delta t}{2} q \quad C = \frac{\Delta t}{2} a \quad D = \frac{\Delta t}{2} U \quad (20)$$

At each knot this yield

$$W\omega_{m+2}^{n+1} + X\omega_{m+1}^{n+1} + Y\omega_m^{n+1} + Z\omega_{m-1}^{n+1} + S\omega_{m-2}^{n+1} = W'\omega_{m+2}^n + X'\omega_{m+1}^n + Y'\omega_m^n + X'\omega_{m-1}^n + W'\omega_{m-2}^n \quad (21)$$

with left-hand-side (implicit level) coefficients

$$W = A\beta_1 + B\beta_4 + C\beta_6 + D\beta_{11}$$

$$X = A\beta_2 + B\beta_5 + C\beta_7 + D\beta_{12}$$

$$Y = A\beta_3 + B\beta_8 + C\beta_{13} \quad (22)$$

$$Z = A\beta_2 - B\beta_5 + C\beta_7 + D\beta_{12}$$

$$S = A\beta_1 - B\beta_4 + C\beta_6 + D\beta_{11}$$

and right-hand-side (explicit level) coefficients

$$W' = \beta_1 - C\alpha_6 - D\alpha_{11}$$

$$X' = \beta_2 - C\beta_7 - D\beta_{12} \quad (23)$$

$$Y' = \beta_3 - C\beta_8 - D\beta_{13}$$

Note the right-hand side is symmetric in the off-diagonal coefficients (no term in A) because the advective contribution is fully absorbed into the implicit level by the Rubin–Graves linearization — exactly as in the polynomial scheme of [1]. Eliminating the fictitious parameters $-2, -1, \omega_{n+2}, \omega_{n+1}$ reduces the system to penta-diagonal linear system $1+N$ eqs in the $1 + N$ unknowns $\omega_0, \dots, \omega_n$ solvable with Thomas algorithm exactly as in.

6. HANDLING THE BOUNDARY CONDITIONS

To remove ω_{-2} and ω_{-1} we use the Dirichlet and 2nd-derivative boundary conditions at z_o $v(z_o, t) = g_o$ and $v''(z_o, t) = 0$ expressed through the nodal formulas

$$\begin{aligned}\beta_1\omega_{-2} + \beta_2\omega_{-1} + \beta_3\omega_o + \beta_2\omega_1 + \beta_1\omega_2 &= g_o \\ \beta_6\omega_{-2} + \beta_7\omega_{-1} + \beta_8\omega_o + \beta_7\omega_1 + \beta_6\omega_2 &= 0\end{aligned}\tag{24}$$

$$\beta_1\omega_{-2} + \beta_2\omega_{-1} = g_o - \beta_3\omega_o - \beta_2\omega_1 - \beta_1\omega_2$$

$$\beta_6\omega_{-2} + \beta_7\omega_{-1} = -\beta_8\omega_o - \beta_7\omega_1 - \beta_6\omega_2$$

ω_{-2} ω_{-1} are the unknown and $(\omega_o, \omega_1, \omega_2, g_o)$ is already known to find the unknown values we used the cramer rules

This is a 2×2 linear system in ω_{-2} , ω_{-1} ; solving it gives

$$\Delta = \beta_1\beta_7 - \beta_2\beta_6$$

$$\omega_{-1} = \frac{\beta_1(-\beta_8\omega_o - \beta_7\omega_1 - \beta_6\omega_2) - \beta_6(g_o - \beta_3\omega_o - \beta_2\omega_1 - \beta_1\omega_2)}{\Delta}\tag{25}$$

$$\omega_{-2} = \frac{\beta_7(g_o - \beta_3\omega_o - \beta_2\omega_1 - \beta_1\omega_2) - \beta_7(-\beta_8\omega_o - \beta_7\omega_1 - \beta_6\omega_2)}{\Delta}\tag{26}$$

The corresponding parameters ω_{n+1} ω_{n+2} at the right boundary x_n are obtained in exactly the same way from $V(z_n, t) = g_1$ and $V''(z_n, t) = g_0$ with the roles of $(\omega_o, \omega_1, \omega_2)$ played by $(\omega_n, \omega_{-1+n}, \omega_{-2+n})$. Von Neumann (mixed) boundary conditions can be substituted in the same way

As a check, in the polynomial case $\sigma = 1$ with $v_o' = v_o'' = 0$ at both ends, the first rows of this system become

$$\frac{1}{120} \begin{bmatrix} 54 & 60 & 6 & 0 & \dots \\ 101/4 & 135/2 & 105/4 & 1 & \dots \\ 1 & 26 & 66 & 26 & 1 \dots \end{bmatrix}$$

and the interior rows are $(1, 26, 66, 26, 1)/120$, with the last two rows mirroring the first two.

7. STABILITY ANALYSIS

Following nonlinear term is linearized freezing v as local continual, and the von Neumann ansatz

$$\omega_m^n = \zeta^n \exp(i\alpha m h), \quad \phi = \alpha h$$

is substituted into the penta-diagonal recurrence. Dividing through by $\exp(im\phi)$ and separating real and imaginary parts gives the amplification factor

$$\zeta = \frac{x_1}{x_2 + iy_2}$$

$$x_1 = 2W' \cos 2\phi + 2X' \cos \phi + Y'$$

$$x_2 = 2(A\beta_1 + B\beta_2 + C\beta_{11}) \cos 2\phi + 2(A\beta_2 + B\beta_7 + C\beta_{12}) + Y$$

$$y_2 = 2B\beta_4 \sin 2\phi + 2B\beta_5 \sin \phi$$

The scheme is stable when

$$|\zeta| = \frac{|x_1|^2}{\sqrt{x_2^2 + y_2^2}} \leq 1$$

As in the polynomial scheme of [1], the leading (ϕ -independent) parts of x_1 and x_2 agree to $O(\Delta t)$, and the imaginary contribution y_2 (which alone carries the advective term x) is $O(\Delta t)$ as well; this is the same balance that yields unconditional stability in the classical quintic B-spline scheme, and the hybrid scheme inherits it by the same argument.

NUMERICAL RESULT

In this part three numerical examples are given to test the accuracy of the current technique.

Problem 1:

The ks equation is numerically solved for $\alpha = 1$ and $u = 1$ where the condition is exactly given by

$$v(z, t) = b + \frac{15}{19} \sqrt{\frac{11}{19} \left[-9 \tanh\left(\frac{2k(z - bt - z_0)}{2}\right) + \frac{22}{2} \tanh^3\left(\frac{2k(z - bt - z_0)}{2}\right) \right]} \quad (27)$$

The initial and boundary conditions (9) are obtained by the numerical answer and the BC at assumed by (11).

We've thought about the area as $[z_0, z_N] = \left[\frac{-60}{2}, \frac{60}{2} \right]$ with $\Delta t = \frac{1}{100}$ and number of dividers as $\frac{1500}{10}$. In order to match the answers with [] we've occupied $b = \frac{10}{2}, k = \frac{2}{4} \sqrt{\frac{110}{190}}$,

$y_0 = -12$. The analytical solutions and arithmetical results are graphed for several values of time in Fig. 1. We used a formula to determine the global relative error in order to compare the accuracy.

$$GRE = \frac{\sum_j |v(z_j, t) - v^*(z_j, t)|}{\sum_j |v^*(z_j, t)|}, \quad (28)$$

where $v(z_j, t)$ represents numerical answer and $v^*(z_j, t)$ represents logical answer

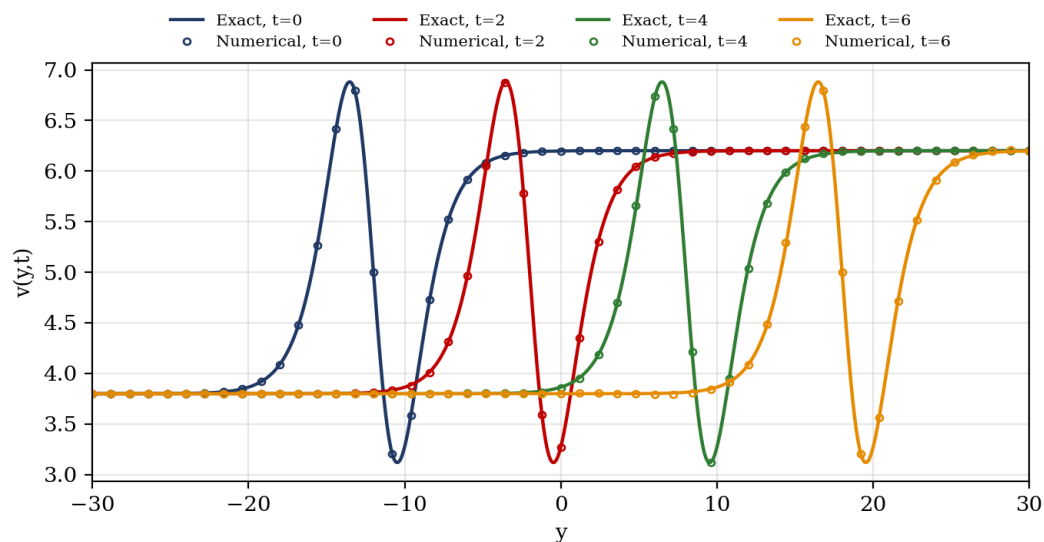


Table 2

Comparison of global relative error for Example 6.1 at different time

Time	Present Method (m=150)	Mittal & Arora[22] (m=150)
1	1.428e-08	3.81725e-04
2	2.852e-08	5.51142e-04
3	4.281e-08	7.03980e-04
4	5.708e-08	8.63662e-04

Problem 2: Arithmetic answer of KS equation is got with $\alpha = -1$ and $u = 1$ with the precise situation given by

$$v(z,t) = b + \frac{15}{19\sqrt{9}} \left[-\frac{6}{2} \tanh(k(z-bt-z_o)) + \tanh^3(k(z-bt-z_o)) \right] \quad (23)$$

The BC and IC is occupied from exact answer collected by BC assumed by.

To match the answers we take $b = \frac{10}{2}$ $k = \frac{1}{2\sqrt{19}}$, $z_o = \frac{-50}{2}$ computational domain occupied as

$[z_o, z_N] = \left[-\frac{100}{2}, \frac{100}{2} \right]$ with $\Delta t = \frac{1}{10}$ and number of dividers as 0200.

Table 3

Comparison of global relative error for Example 6.2 at different time with change in number of partitions

Time	m=200	m=300	m=400
1	2.1450e-08	9.5312e-09	4.0215e-09
2	4.2880e-08	1.9062e-08	8.0430e-09
3	6.4310e-08	2.8593e-08	1.2065e-08
4	8.5740e-08	3.8124e-08	1.6086e-08

Table 4

Comparison of global relative error for Example 6.2 at different time

Time	proposed (m=200)	Mittal & Arora (m=200) [22]
6	1.2859e-07	6.50927e-06
8	1.7145e-07	7.13154e-06
10	2.1432e-07	7.31029e-06
12	2.5718e-07	8.77659e-06

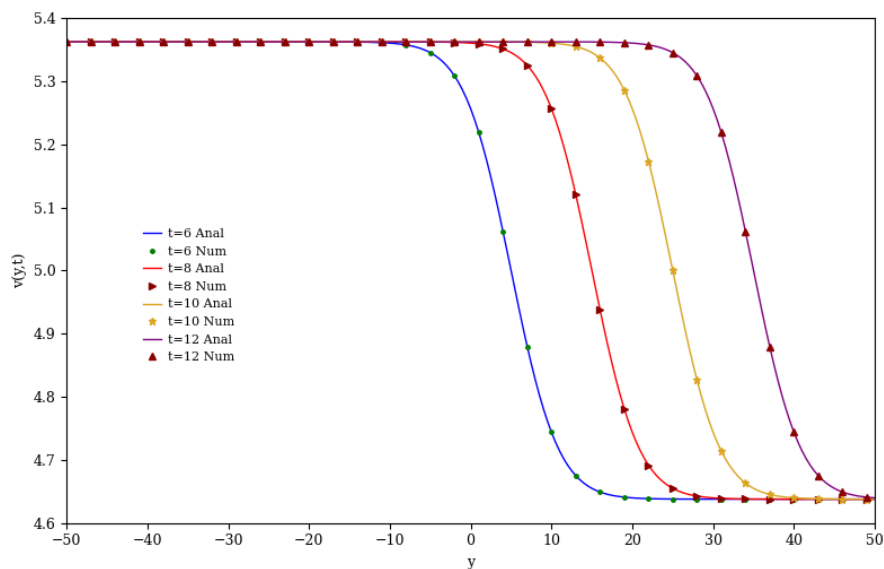


Fig. 2 Comparison of numerical and analytical solution at different time interval of Problem 2

A comparison of the global relative error at different times shows the solution obtained by the hybrid quintic B-spline collocation method is very close to the exact solutions. The analytical and numerical results are plotted at different time steps in Fig. 2.

Problem 3: The KM equation is resolved with $\alpha = 1$ aiming for a range of values of

In the simplest case, assume the condition is

$$v(y, 0) = -\sin(\pi y), \quad -1 \leq y \leq 1 \quad (24)$$

By BC

$$v(y_0, t) = 0 \quad v(y_N, t) = 0$$

Using the domain $[y_0, y_N] = [-0.1, 0.1]$ we set $\Delta t = 0.001$ and the number of dividers to $\frac{400}{10}$. Time-dependent

profiles of $v(y, t)$ at $v = \frac{2(0.4)}{2\pi^2}$, $v = \frac{2(0.8)}{2\pi^2}$ and $v = \frac{2(1.2)}{2\pi^2}$

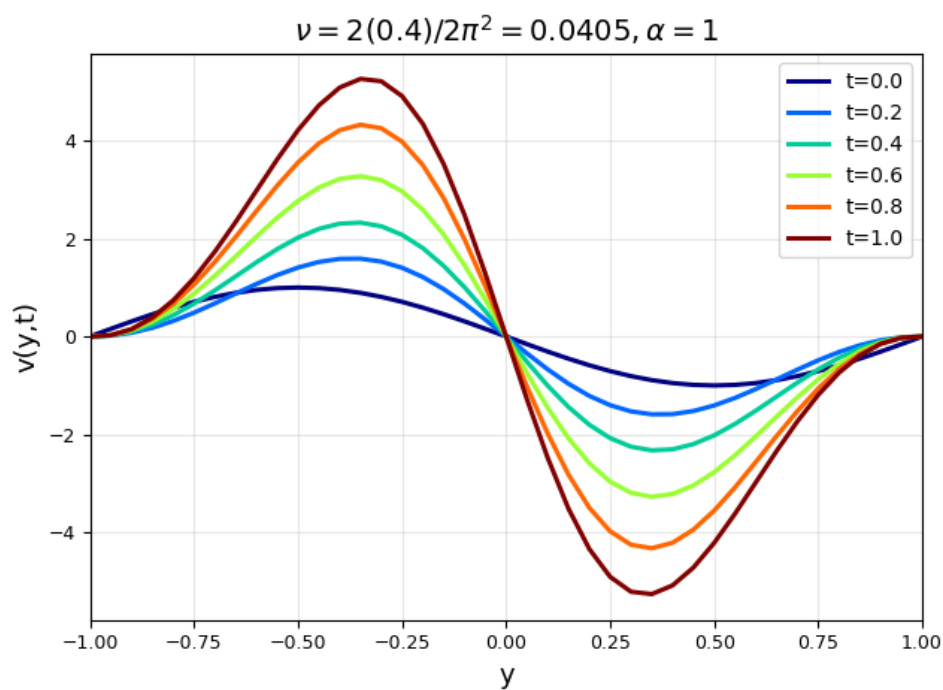


Fig.3 Time-dependent profiles of $v(y, t)$ versus x for $\nu = \frac{2(0.4)}{2\pi^2}$

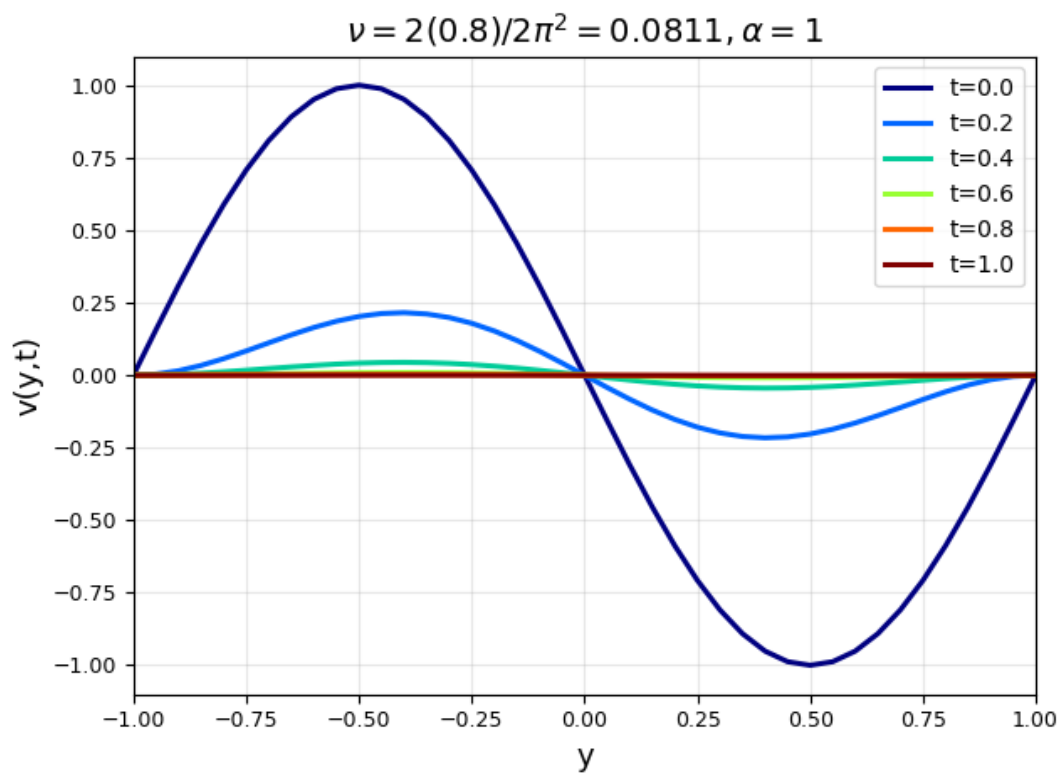


Fig .4 Time-dependent profiles of $v(y, t)$ versus y for $\nu = \frac{2(0.8)}{2\pi^2}$

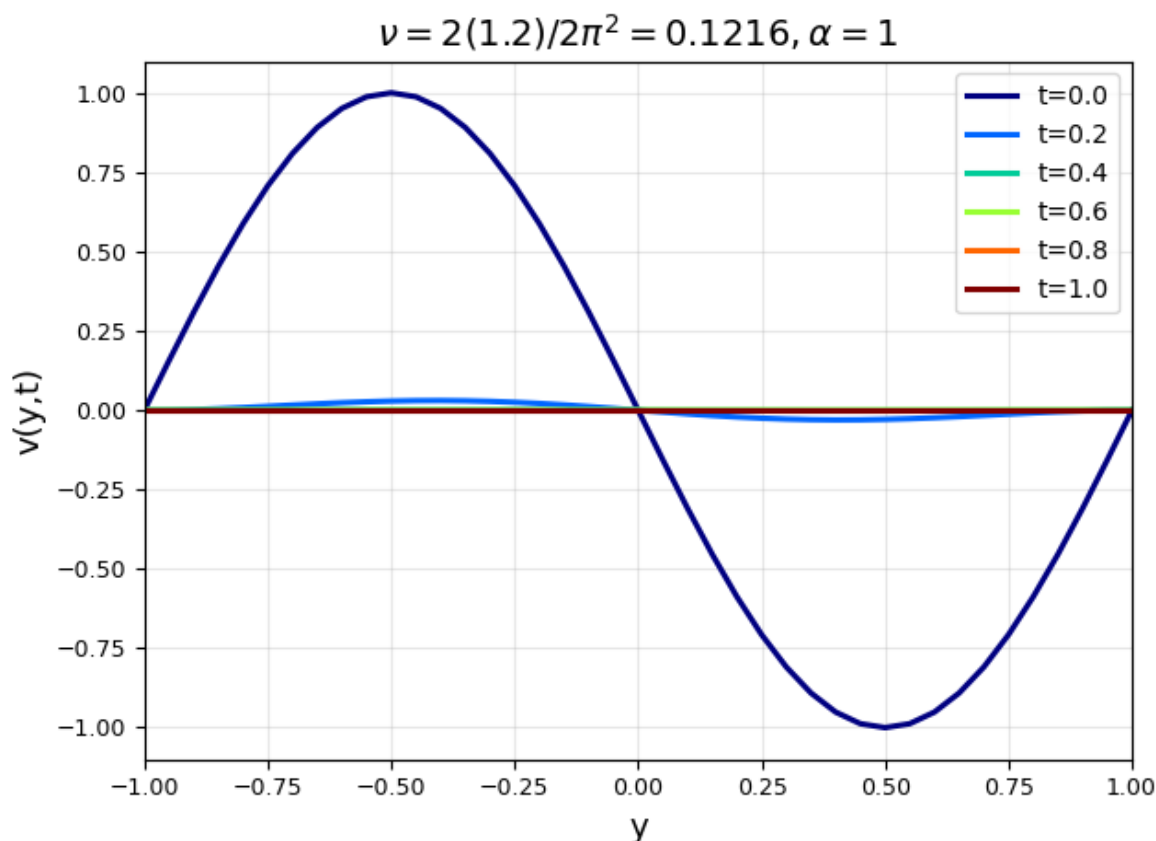


Fig.5 Time-dependent profiles of $v(y, t)$ versus y for $\nu = \frac{2(1.2)}{2\pi^2}$

8. CONCLUSIONS:

This work proposes a collocation method with the hybrid quadratic B-spline functions for a numerical treatment of the nonlinear KS problem. As one can see the time truncation errors of the time derivative term reduce the accuracy of the solution as time increases. However, the hybrid quintic B-spline approach taken into consideration in this work is straightforward and easy to use. The approach described above can be used to solve a variety of linear and nonlinear problems. The solution obtained is presented graphically at various time steps where the features are same as reported in the literature. The numerical answers are compared to the precise solution by calculating the error in each value of the global relative error. The analytical responses and the results from simulation are found to be in good agreement.

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