

IMPROVEMENT OF ACTIVE DISTURBANCE REJECTION CONTROL PERFORMANCE IN THE TARGET-INDICATING GIMBAL SYSTEM USING THE M-TH ORDER EXTENDED STATE OBSERVER WITH FREQUENCY ESTIMATION

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Article Info

Abstract



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In this paper, by active disturbance rejection control (ADRC) using m-th order extended state observer (ESO) with frequency estimation, the method to improve disturbance rejection property is improved and it is applied for satisfying high-accuracy control property in the target-indicating gimbal system. In this paper, error coefficient of ADRC using m-th order ESO is analyzed for control object expressed as canonical form. In addition, the frequency estimator to estimate unknown frequency is combined and rejection property is analyzed and evaluated to constant disturbance, lamp disturbance parabolic disturbance, and harmonic disturbance (single or multiple frequency disturbance).

Keywords:

Frequency Estimation, Active Disturbance Rejection Control (ADRC), Extended State Observer (ESO), Disturbance Rejection

Introduction

Improving the disturbance rejection property of target-indicating gimbal system is important for raising control accuracy [1, 20-23]. Recently, ADRC rejecting the disturbance as 'Whole' is concerned by control system designers [2-5].

ADRC is researched and applied to express to canonical form having integrator chain structure by compensation of ESO for 'Whole disturbance' [6-15].

In ADRC that researched recently, uses one-order ESO.

In ADRC using one-order ESO, the disturbance rejection performance is raised by increasing the natural angle frequency of observer. $(n+1)$ -order ADRC has high rejection performance for constant disturbance but the rejection performance is limited to ramp disturbance, parabolic disturbance and harmonic disturbance.

Thus, for raising the performance of one-order ESO, the disturbance rejection performance is raised by combining nonlinear function such as fal-function, sat function and sigmoid function and configuring the nonlinear observer.

But in case of using nonlinear ESO, the observer performance is raised in range of linearity by multiply observer transfer coefficient by linear function transfer coefficient, but in saturation area, the disturbance rejection performance of observer is dropped.

In paper [16], in case of the disturbance frequency is known, disturbance rejection control method by ADRC is introduced.

In ADRC subjected to the harmonic disturbance, it is difficult to raise disturbance rejection performance in case of not knowing the angle frequency of harmonic disturbance [18-19].

In paper [18-19], to resonate and adjust output amplitude of shaft by ADRC, actuation of actuating shaft of MEMS is shown. Paper [17] shows if the resonance frequency is selected properly, sine disturbance is rejected and in static state, signal tracking of ideal reference can be performed. The $(n+1)$ -order ADRC system has one-order astaticism for disturbance and $(n+m)$ -th order ADRC system has m -order astaticism for disturbance.

Eventually, $(n+1)$ -order ADRC can make disturbance error be zero for constant disturbance and it can't make other disturbance error be zero theoretically.

Also, it is difficult to find data about disturbance rejection performance analysis of frequency estimating-type $(n+m)$ -th order ADRC.

Thus, in this paper, the method to improve harmonic disturbance rejection performance using frequency estimating-type $(n+m)$ -th order ADRC in ADRC system is researched.

Firstly, this paper shows that for aperiodic disturbance, ADRC system having m -order ESO rejects constant disturbance, ramp disturbance and parabolic disturbance perfectly according to m .

Secondly, this paper shows that in case of using frequency estimating-type ESO, ADRC system rejects harmonic disturbance mostly for frequency harmonic disturbance and the harmonic disturbance rejection performance improved remarkably for multiple frequency harmonic disturbance.

The paper consists of following sections.

In section 1, identification model of target-indicating gimbal equipment is discussed.

In section 2, the model of (n+m)-th order ADRC system is made and disturbance rejection method of frequency estimating-type (n+m)-th order ADRC is researched.

In section 3, the stability of frequency estimating-type (n+m)-th order ADRC is discussed.

In section 4, the simulation result is analyzed for constant disturbance, ramp disturbance, parabolic disturbance and harmonic disturbance (single and multiple frequency disturbance), and the test result of target-indicating gimbal equipment is analyzed.

In section 5, conclusion is given.

In control system using ADRC, using frequency estimating-type (n+m)-th order ADRC, it is confirmed that the disturbance rejection ability can be raised higher.

1. MODEL IDENTIFICATION OF GIMBAL EQUIPMENT AND PROPOSED PROBLEM

It is very complex and difficult to get accurate mathematical model of target-indicating gimbal equipment analytically. Therefore, the mathematical model of target-indicating gimbal equipment is obtained by identification method.

Schematic diagram of target-indicating gimbal equipment is given below.

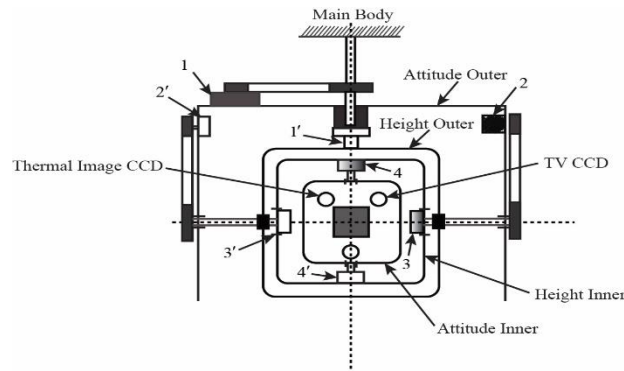


Figure 1: Schematic diagram of target-indicating gimbal equipment.

In the Fig. 1, 1 and 1' are azimuth outer motor and encoder, 2 and 2' are elevation outer motor and encoder, 3 and 3' are elevation inner motor and 4 and 4' are attitude inner motor and encoder.

The model of control plant including current driver, torque motor and gimbal device is expressed as follows

$$W(s) = \frac{\omega(s)}{U(s)} = \frac{K_I}{T_I s + 1} \quad (1)$$

where $W(s)$ is transfer function of object, $\omega(t)$ (1/s) is rotating speed of torque motor, $u(t)$ is input voltage to driver, K_I and T_I are transfer coefficient and time constant respectively.

For identification of control object, sample time T_0 is 0.001s, identification error is made to be satisfied with the output error signal $e(t)$ less than 0.1mrad.

For confirmation of plant model, frequency response signal having 1~100Hz of frequency f is given to the input and the model identification is performed to make the output of experimental data equal to one of theoretical model.

Input/output diagram adopted in real plant is given in the following figure.

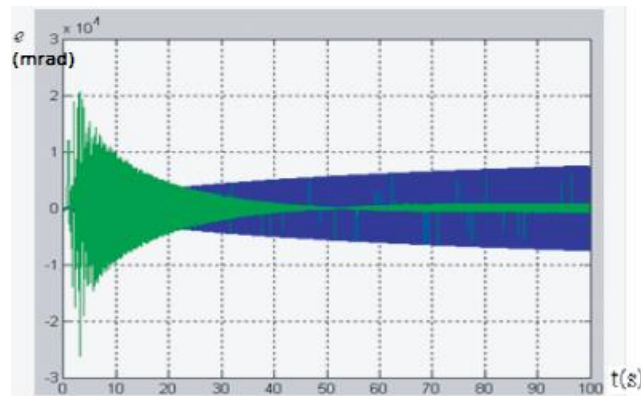


Figure 2: Input and output diagram adapted in real plant. Identification result curves are shown in the following figure.

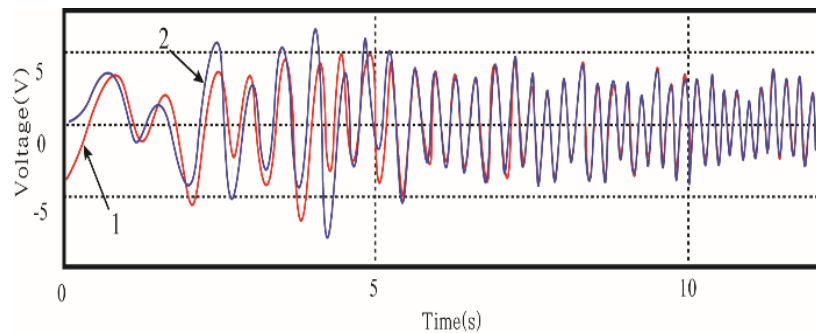


Figure 3: Identification data.

Identification data confirmed of target-indicating gimbal equipment is given in Table 1.

Table 1: Identification data of target-indicating equipment

№	Item	Mark	Unit	Value
1	Resistance	R	Ω	7.3
2	Inductance	L	H	$1.9 \cdot 10^{-3}$
3	Electromotive coefficient	C_e	Vs	0.15
4	Torque coefficient	C_m	Nm / A	0.15
5	Inertia torque	J	Kgm^2	4.5×10^{-3}
6	Transfer coefficient	k_t		102564
7	Time constant	T_t	s	3.9×10^{-4}

External disturbance actuating on target-pointing four-ring double-axis gimbale equipment is as follows,

$$d = M_0 + M(\omega) + M_t \quad (2)$$

where $M_0 = 0.001 Nm$, $M_\omega = -0.005 \sin(\omega)$, $M_t = 0.39 \sin(1.33t)$. Then as disturbance rejection performance test result of target-indicating gimbal equipment, in case of not estimating frequency, the value is $2.1mrad$, which isn't satisfied with technical requirement- $0.2 mrad$ below.

2. DISTURBANCE REJECTION METHOD IN THE (N+M)-TH ORDER ADRC SYSTEM WITH FREQUENCY ESTIMATION

In this paper, in Target-indicating gimbal system, (n+m)-th order ADRC method is going to be researched to reject lamp disturbance, parabolic disturbance and unknown disturbance.

To simplify the problem, the model of control plant is denoted as follows

$$\left. \begin{aligned} \dot{x}_i(t) &= x_{i+1}(t); (i = \overline{1, n-1}) \\ \dot{x}_n(t) &= F(x_1(t), x_2(t), \dots, x_n(t)) + b_0 u(t) + d(t) \end{aligned} \right\} t \geq t_0$$

$$y(t) = x_1(t)$$

$$x_i(0) = 0$$
(3)

where $X(t)(X(t) \in R^n)$ is state vector, $y(t)(y(t) \in R)$ is observable output signal and $u(t)(u(t) \in R)$ is control input.

Assumption1: Known nominal value b_0 is the same as definitive parameter.

Assumption2: Eq. (1) can be stabilized by feedback control.

Assumption3: Eq. (1) is denoted as the canonical form of linear chain integrator.

Assumption4: Extended order of ESO is set as m.

Assumption5: From the purpose of paper, reference signal is set as $r(t) = 0$ and external disturbance $d(t)$ is denoted as follows with considering extended order from paper [17].

$$\begin{aligned} d(t) &= d_1(t) + d_2(t) \\ d_1 &= \sum_{i=1}^m \alpha_{i-1} t^{i-1}; \quad |d_1(t)| \leq \bar{d}_1; \\ d_2 &= A \sin \omega_d t \quad |d_2(t)| \leq \bar{d}_2 \end{aligned} \quad (4)$$

Here $d_1(t)$ is aperiodic disturbance, α_{i-1} is maximum value of derived function and m-order derived function of $d_1(t)$ is zero.

And $d_2(t)$ is harmonic disturbance and $A(|A| \leq \bar{A})$, $\omega_d(\omega_d \leq \bar{\omega}_d)$ is amplitude and angle frequency. α_{i-1} , A , ω_d are uncertain quantities.

Assumption 6: In the Eq. (3), nonlinear internal uncertainty and external disturbance have superposability and lump disturbance is as follows.

$$w[F(x_1(t), x_2(t), \dots, x_n(t)), d(t)] = w_F + w_d \quad (5)$$

Here let's assume as follows.

$$\begin{aligned} w_F &= [F(x_1(t), x_2(t), \dots, x_n(t))] \leq [w_F] \\ w_d &= d(t) \leq w_d \end{aligned} \quad (6)$$

In Eq. (3), if extended state variable $x_{n+1}(t)$ is denoted as follows,

$$x_{n+1}(t) = w[F(x_1(t), x_2(t), \dots, x_n(t)), d(t)] = h(t) \quad (7)$$

then,

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t); \quad (i = \overline{1, n-1}) \\ \dot{x}_n(t) &= x_{n+1}(t) + b_0 u(t) \end{aligned} \quad \left. \vphantom{\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t); \quad (i = \overline{1, n-1}) \\ \dot{x}_n(t) &= x_{n+1}(t) + b_0 u(t) \end{aligned}} \right\} t \geq t_0$$

$$\begin{aligned} y(t) &= x_1(t) \\ x_i(0) &= 0 \end{aligned} \quad (8)$$

Deriving Eq. (7) by m times and combining to Eq. (9), it can be denoted as canonical form m-order extended state model.

$$\left. \begin{aligned} \dot{x}_i(t) &= x_{i+1}(t); i = \overline{1, n-1} \\ \dot{x}_n(t) &= x_{n+1}(t) + b_0 u(t); \\ \dot{x}_{n+j}(t) &= x_{n+j+1}(t); j = \overline{1, m-1}; \\ \dot{x}_{n+m}(t) &= h^{(m)}(t) \\ y(t) &= x_1(t), \end{aligned} \right\} \quad (9)$$

Here $h^{(m)}$ is m-order derived function of $h(t)$.

2.1. LINEARIZATION BY THE m^{th} ORDER EXTENDED STATE OBSERVER

Considering the case of $d(t) = 0$ in Eq. (2), the extended state of Eq. (5) is as follows.

$$x_{n+1}(t) = w_F = h_F(t) \quad (10)$$

From Eq. (10), Eq. (9) is as follows.

$$\left. \begin{aligned} \dot{x}_i(t) &= x_{i+1}(t); i = \overline{1, n-1} \\ \dot{x}_n(t) &= x_{n+1}(t) + b_0 u(t); \\ \dot{x}_{n+j}(t) &= x_{n+j+1}(t); j = \overline{1, m-1}; \\ \dot{x}_{n+m}(t) &= h_F^{(m)}(t) \\ y(t) &= x_1(t), \end{aligned} \right\} \quad (11)$$

the $(n+m)$ -th order extended state observer can be denoted as follows for Eq. (11).

$$\left. \begin{aligned} \dot{\hat{x}}_i(t) &= \hat{x}_{i+1}(t) + \beta_i(y(t) - \hat{y}(t)); (i = \overline{1, n-1}) \\ \dot{\hat{x}}_n(t) &= \hat{x}_{n+1}(t) + b_0 u(t) + \beta_n(y(t) - \hat{y}(t)) \\ \dot{\hat{x}}_{n+j}(t) &= \hat{x}_{n+j+1}(t) + \beta_{n+j}(y(t) - \hat{y}(t)); (j = \overline{1, m-1}) \\ \dot{\hat{x}}_{n+m}(t) &= \beta_{n+m}(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= \hat{x}_1(t) \end{aligned} \right\} t \geq t_0 \quad (12)$$

Here $\hat{x}_i(t)$ is observer state vector, $\hat{y}(t)$ is observer output signal and β_i is observer gain satisfying characteristic

polynomial $s^{n+m} + \sum_{i=1}^{n+m} \beta_i s^{n+m-i}$ for Hurwitz stability.

If it is possible to observe disturbance of control plant from ESO, control law is denoted as follows.

$$u(t) = -\frac{\hat{x}_{n+1}(t)}{b_0} + u_0(t) \quad (13)$$

Substituting Eq. (13) for Eq. (11) and considering $x_{n+1}(t) = \hat{x}_{n+1}(t)$, the equation is as follows.

$$\left. \begin{aligned} \dot{x}_i(t) &= x_{i+1}(t); i = \overline{1, n-1} \\ \dot{x}_n(t) &= b_0 u_0(t); \\ \dot{x}_{n+j}(t) &= x_{n+j+1}(t); j = \overline{1, m-1}; \\ \dot{x}_{n+m}(t) &= h_F^{(m)}(t) \\ y(t) &= x_1(t), \end{aligned} \right\} \quad (14)$$

Eventually it can be seen that if $x_{n+1}(t) = \hat{x}_{n+1}(t)$, Eq. (14) is linearized for nonlinear uncertainty.

2.2. DETERMINATION OF CONTROL LAW $u_0(t)$

In Eq. (13), considering assumption 5, let's assume that control law $u_0(t)$ is determined as follows.

$$u_0(t) = - \sum_{i=1}^n k_i x_i(t) \quad (15)$$

Here k_i is controller coefficient.

Thus, control law of ADRC is obtained from Eq. (11) as follows.

$$u(t) = - \frac{\hat{x}_{n+1}(t)}{b_0} - \sum_{i=1}^n k_i x_i(t) \quad (16)$$

2.3. ERROR COEFFICIENT ANALYSIS OF ADRC WITH THE m^{th} ORDER ESO

In Eq. (3), nonlinear uncertainty is rejected by Eq. (11), and let's consider the case of $d_2(t) = 0$ i.e. $d(t) = d_1(t)$.

Then the extended state of Eq. (7) is as follows.

$$x_{n+1}(t) = w_d = h_{d1}(t) \quad (17)$$

According to Eq. (9), Eq. (17) is as follows.

$$\left. \begin{aligned} \dot{x}_i(t) &= x_{i+1}(t); i = \overline{1, n-1} \\ \dot{x}_n(t) &= x_{n+1} + b_0 u(t); \\ \dot{x}_{n+j}(t) &= x_{n+j+1}(t); j = \overline{1, m-1}; \\ \dot{x}_{n+m}(t) &= h_{d1}^{(m)}(t) \\ y(t) &= x_1(t), \end{aligned} \right\} \quad (18)$$

According to Eq. (16), Eq. (18) is as follows.

$$\left. \begin{aligned} \dot{x}_i(t) &= x_{i+1}(t); i = \overline{1, n-1} \\ \dot{x}_n(t) &= b_0 u(t); \\ \dot{x}_{n+j}(t) &= x_{n+j+1}(t); j = \overline{1, m-1}; \\ \dot{x}_{n+m}(t) &= h_{d1}^{(m)}(t) \\ y(t) &= x_1(t), \end{aligned} \right\} \quad (19)$$

Eventually, it can be seen that Eq. (19) rejects polynomial disturbance- $d(t) = d_1(t)$, if $x_{n+1}(t) = \hat{x}_{n+1}(t)$.

Performing the error coefficient analysis of ADRC, having m-order extended state observer, the error of system is denoted as follows.

$$e(t) = r(t) - y(t) \quad (20)$$

Transforming Eq. (18) and Eq. (20) into Laplace transform and finding $E(s)$, the equation is as follows.

$$E(s) = \frac{\hat{X}_{n+1}(s) - W(s)}{\left(s^n + b_0 \sum_{i=1}^n k_i s^{i-1} \right)} \quad (21)$$

Here $E(s)$ is Laplace transform of $e(t) = r(t) - y(t)$, $r(t)(r(t)=0)$ is target signal and $\hat{X}_{n+1}(s)$, $W_{d1}(s)$ is Laplace transform of $X_{n+1}(t)$, $d_1(t)$.

The observer error is denoted as follows.

$$\hat{e}(t) = y(t) - \hat{y}(t) = r(t) - e(t) - \hat{y}(t) \quad (22)$$

Meanwhile, transforming Eq. (12) and Eq. (22) into Laplace transformation, observer error is denoted as follows.

$$\hat{E}(s) = \frac{-(s^n + b_0 \sum_{i=1}^n k_i s^{i-1})E(s)}{s^n + \sum_{i=1}^n \beta_i s^{n-i}} \quad (23)$$

Also, in Eq. (12), finding extended state $\hat{X}_{n+1}(s)$, the equation is as follows.

$$\hat{X}_{n+1}(s) = \frac{\left(\sum_{j=1}^m \beta_{n+j} s^{m-j} \right)}{s^m} \hat{E}(s) \quad (24)$$

Substituting Eq. (23) for Eq. (24),

$$\hat{X}_{n+1}(s) = \frac{\left(\sum_{j=1}^m \beta_{n+j} s^{m-j} \right)}{s^m} \cdot \frac{-(s^n + b_0 \sum_{i=1}^n k_i s^{i-1})E(s)}{s^n + \sum_{i=1}^n \beta_i s^{n-i}} \quad (25)$$

Substituting Eq. (25) for Eq. (21) and arrange one, error model of ADRC system having m-order ESO is obtained as follows.

$$E(s) = \frac{s^m(s^n + \sum_{i=1}^n \beta_i s^{n-i})W(s)}{s^m(s^n + \sum_{i=1}^n \beta_i s^{n-i})(s^n + b_0 \sum_{i=1}^n k_i s^{i-1}) + (\sum_{j=1}^m \beta_{n+j} s^{m-j})(s^n + b_0 \sum_{i=1}^n k_i s^{i-1})} \tag{26}$$

Here, $E(s)$ is error for the disturbance- $W_{d1}(s)$.

In Eq. (26), the error transfer function between lump disturbance-error is as follows.

$$\Phi(s) = \frac{s^m \cdot}{s^m \left(s^n + \sum_{i=1}^n \beta_i s^{n-i} \right) \left(s^n + b_0 \sum_{i=1}^n k_i s^{i-1} \right) + \left(s^n + \sum_{i=1}^n \beta_i s^{n-i} \right) \left(\sum_{j=1}^m \beta_{n+j} s^{m-j} \right)} \tag{27}$$

In Eq. (27), error coefficient is denoted as follows.

$$C_i = \frac{\partial}{\partial s} \Phi(s) \Big|_{s=0} ; \quad i = \overline{0, m} \tag{28}$$

Here, C_i is error coefficient and $\Phi(s)$ is transfer function from disturbance to error.

Table 2: Error Coefficient Table of ADRC According to Extended State Order

Plant \	n=1, m=1	n=1, m=2	n=1, m=1
C_{0d}	0	0	0
C_{1d}	$\frac{\beta_1}{f_1 \beta_2}$,	0	0
C_{2d}	$\neq 0$	$\frac{2\beta_1}{f_1 \beta_3}$	0
C_{3d}	$\neq 0$	$\neq 0$	$\frac{3\beta_1}{f_1 \beta_4}$

From Table 2, (n+m)-th order ADRC system is transferred to m^{th} order astaticism system for disturbance according to extended state order-m and is able to make error be zero for (m-1)-th order disturbance. But from paper [2-3], (n+1)-th order ADRC using linear and nonlinear observer has one-order astaticism. Thus, it is impossible to make error be zero for constant speed disturbance and acceleration disturbance excluding constant disturbance.

2.4. HARMONIC DISTURBANCE REJECTION METHOD BY THE 2ND ORDER ESO WITH FREQUENCY ESTIMATION

In Eq. (3), nonlinear uncertainty is rejected by Eq. (11).

If $d_1(t) = 0$, considering the case of $d(t) = d_2(t)$,

then, Eq. (5) is denoted as follows.

$$x_{n+1}(t) = w_d = h_{d2}(t) = A \sin \omega_{d2} t \tag{29}$$

Deriving Eq. (29) two times continuously,

$$\left. \begin{aligned} h_{d2}(t) &= A \sin \omega_d t = x_{n+1} \\ \dot{h}_{d2}(t) &= A \omega_d \cos \omega_d t = x_{n+2} \\ \ddot{h}_{d2}(t) &= -A \omega_d^2 \sin \omega_d t = -\omega_d^2 h_{d2}(t) = x_{n+3} \\ h_{d2}^{(i)}(t) &= -\omega_d^2 h_{d2}^{(i-2)}; \quad i = \overline{3, m} \end{aligned} \right\} \quad (30)$$

Substituting Eq. (30) for Eq. (8),

$$\left. \begin{aligned} \dot{x}_i(t) &= x_{i+1}(t); i = \overline{1, n-1} \\ \dot{x}_n(t) &= x_{n+1}(t) + b_0 u(t); \\ \dot{x}_{n+i}(t) &= x_{n+i+1}(t); \quad j = \overline{1, m-1} \\ \dot{x}_{n+m}(t) &= -\omega_d^2 x_{n+m-2}(t) \\ y(t) &= x_1(t), \end{aligned} \right\} \quad (31)$$

If disturbance frequency for estimation is $\theta = -\omega_d^2$, Eq. (29) is as follows.

$$\left. \begin{aligned} \dot{x}_i(t) &= x_{i+1}(t); i = \overline{1, n-1} \\ \dot{x}_n(t) &= x_{n+1}(t) + b_0 u(t); \\ \dot{x}_{n+i}(t) &= x_{n+i+1}(t); \quad j = \overline{1, m-1} \\ \dot{x}_{n+m}(t) &= \theta \cdot x_{n+m-2}(t) \\ y(t) &= x_1(t), \end{aligned} \right\} \quad (32)$$

ESO for Eq. (32) is as follows.

$$\left. \begin{aligned} \dot{\hat{x}}_i(t) &= \hat{x}_{i+1}(t) + \beta_i(y(t) - \hat{y}(t)); \quad i = \overline{1, n-1} \\ \dot{\hat{x}}_n(t) &= \hat{x}_{n+1}(t) + \beta_n(y(t) - \hat{y}(t)) + b_0 u(t) \\ \dot{\hat{x}}_{n+i}(t) &= \hat{x}_{n+i+1}(t) + \beta_{n+i}(y(t) - \hat{y}(t)); \quad j = \overline{1, m-1} \\ \dot{\hat{x}}_{n+m}(t) &= \hat{\theta} \cdot \hat{x}_{n+m-2}(t) + \beta_{n+m}(y(t) - \hat{y}(t)); \\ \hat{y}(t) &= \hat{x}_1(t) \end{aligned} \right\} \quad (33)$$

Obtaining error dynamic characteristic of the system from Eq. (32) and Eq. (33),

$$\left. \begin{aligned} \dot{x}_{ei} &= x_{e(i+1)} - \beta_i x_{e1}; \quad i = \overline{1, n+m-1} \\ \dot{x}_{e(n+m)} &= \theta x_{n+m-2} - \hat{\theta} \hat{x}_{n+m-2} - \beta_{n+m} x_{e1} \end{aligned} \right\} \quad (34)$$

where, $x_{ei} = \hat{x}_i - x_i$ and $\tilde{\theta}$ is parameter estimation error - $(\tilde{\theta} = \hat{\theta} - \theta)$.

Subtracting $\hat{\theta} x_{n+i-2}$ to second term of Eq. (34), arranging Eq. (34) by considering $\tilde{\theta} = \hat{\theta} - \theta$, the result is as follows.

$$\left. \begin{aligned} \dot{x}_{ei} &= x_{e(i+1)} - \beta_i x_{e1}; \quad i = \overline{1, n+m-1} \\ \dot{x}_{e(n+m)} &= \hat{\theta} x_{e(n+m)} + \tilde{\theta} (\hat{x}_{n+m-2} - x_{e(n+m-2)}) - \beta_{n+m} x_{e1} \end{aligned} \right\} \quad (35)$$

Denoting Equation as state model,

$$\dot{\hat{X}} = (A_1 + \hat{\theta}A_2 - LC)\tilde{X} + \tilde{\theta}A_2(\hat{X} - \tilde{X}) \quad (36)$$

Where

$$\tilde{X} = (x_{e1} \ x_{e2} \ \cdots \ x_{e(n+m)})^T, \quad L = (\beta_1 \ \beta_2 \ \cdots \ \beta_{n+m})^T \quad A_1 = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix}_{(n+m) \times (n+m)}, \quad A_2 = \begin{pmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 1 & 0 \end{pmatrix}_{(n+m) \times (n+m)}$$

$$C = (1 \ 0 \ 0 \ \cdots \ 0)_{1 \times (n+m)}$$

To determine estimation law, Lyapunov function is introduced as follows.

$$V = \frac{1}{2} \tilde{X}^T P \tilde{X} + \frac{1}{2\lambda} \tilde{\theta}^2 \quad (37)$$

Here, P is positive value matrix and λ is weight factor.

Deriving frequency estimator from convergence condition of Eq. (37),

$$\begin{aligned} \dot{V} &= \tilde{X}^T P \dot{\tilde{X}} + \tilde{\theta} \dot{\tilde{\theta}} \\ &= \tilde{X}^T (\bar{A}^T P + P \bar{A}) \tilde{X} + \tilde{\theta} (X^T A_2^T P \tilde{X} + \tilde{X}^T P A_2 X + \frac{1}{\lambda} \dot{\tilde{\theta}}) \\ &= -\tilde{X}^T Q \tilde{X} + \tilde{\theta} (X^T A_2^T P \tilde{X} + \tilde{X}^T P A_2 X + \frac{1}{\lambda} \dot{\tilde{\theta}}) \end{aligned} \quad (38)$$

Where

$$\bar{A} = A_1 + \hat{\theta}A_2 - LC, \quad X = \hat{X} - \tilde{X}.$$

In Eq. (38), denoting as follows

$$\bar{A}^T P + P \bar{A} = -Q \quad (39)$$

if positive definite matrix Q , exists, positive definite value matrix P is determined, and setting as follows

$$X^T A_2^T P \tilde{X} + \tilde{X}^T P A_2 X + \frac{1}{\lambda} \dot{\tilde{\theta}} = 0 \quad (40)$$

Then, frequency estimation law is as follows from Eq. (40).

$$\dot{\tilde{\theta}} = -\lambda (X^T A_2^T P \tilde{X} + \tilde{X}^T P A_2 X) \quad (41)$$

In Eq. (41), if $\dot{\tilde{\theta}} = \dot{\hat{\theta}} - \dot{\theta}$ and $\dot{\theta} \approx 0$ according to time variation, it is denoted as follows.

$$\dot{\hat{\theta}} = -\lambda (X^T A_2^T P \tilde{X} + \tilde{X}^T P A_2 X) \quad (42)$$

From Eq. (42), the result is obtained as follows.

Determining $\hat{\theta}$, frequency can be estimated and harmonic disturbance can be rejected effectively.

Comparing this with paper [16], rejection performance for harmonic disturbance can be improved by estimating the frequency.

2.5. ADRC METHOD BY THE m^{th} ORDER ESO WITH FREQUENCY ESTIMATION

Considering frequency estimation term of Eq. (32) and Eq. (33) and denoting extended state model and extended state observer model of control plant, the Equation is as follows

$$\left. \begin{aligned} \dot{x}_i(t) &= x_{i+1}(t); i = \overline{1, n-1} \\ \dot{x}_n(t) &= x_{n+1}(t) + b_0 u; \\ \dot{x}_{n+j}(t) &= x_{n+j+1}(t) + \theta \cdot x_{n+i-2}(t); \quad j = \overline{1, m-1} \\ \dot{x}_{n+m}(t) &= \theta \cdot x_{n+m-2}(t) + h_{d1}^{(m)}(t) \\ y(t) &= x_1(t), \end{aligned} \right\} \quad (43)$$

$$\left. \begin{aligned} \dot{\hat{x}}_i(t) &= \hat{x}_{i+1}(t) + \beta_i(y(t) - \hat{y}(t)); (i = \overline{1, n-1}) \\ \dot{\hat{x}}_n(t) &= \hat{x}_{n+1}(t) + b_0 u(t) + \beta_n(y(t) - \hat{y}(t)) \\ \dot{\hat{x}}_{n+j}(t) &= \hat{x}_{n+j+1}(t) + \beta_{n+j}(y(t) - \hat{y}(t)); (j = \overline{1, m-1}) \\ \dot{\hat{x}}_{n+m}(t) &= \hat{\theta} \cdot \hat{x}_{n+m-2}(t) + \beta_{n+m}(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= \hat{x}_1(t) \end{aligned} \right\} \quad (44)$$

From Eq. (42), if it is possible to estimate frequency and to observe disturbance of control plant from ESO, the disturbance having random frequency can be rejected effectively by ADRC law, Eq. (16).

Eventually, ADRC system having m^{th} order ESO with frequency estimation can reject aperiodic disturbance and harmonic disturbance effectively.

Comparing proposed method with paper [16], rejection performance is improved more for the unknown harmonic frequency disturbance.

3. STABILITY OF THE $(n+m)^{th}$ ORDER ADRC SYSTEM WITH FREQUENCY ESTIMATION

The stability of whole system can be found as follows.

First: The ESO is linear to aperiodic disturbance, so the stability of whole system is related with the stability of ESO.

From Eq. (43) and Eq. (44), error dynamic characteristic is as follows.

$$\left. \begin{aligned} \dot{x}_{e_i}(t) &= x_{e_{i+1}}(t) - \beta_i x_{e1}(t); (i = \overline{1, n-1}), \\ \dot{x}_{e_n}(t) &= \hat{x}_{e_{(n+1)}}(t) - \beta_n x_{e1}(t) \\ \dot{x}_{e(n+j)}(t) &= x_{e(n+j+1)}(t) - \beta_{n+j} x_{e1}(t); \quad j = \overline{1, m-1}; \\ \dot{x}_{e(n+m)}(t) &= h^{(m)}(t) \\ \hat{e}(t) &= x_{e1}(t) \end{aligned} \right\} \quad (45)$$

Here, $x_{e\ i}(t)$ ($x_{e\ i}(t) = x_i(t) + \hat{x}_i(t)$) is error state variable.

Characteristic equation of Eq. (45) is as follows.

$$D_C(s) = s^{n+m} + \sum_{i=1}^{n+m} \beta_i s^{n+m-i} \quad (46)$$

Here, $D_C(s)$ is characteristic equation of observer.

From characteristic polynomial satisfied with Hurwitz condition,

$$D_{0C}(s) = (s + \omega_C)^{n+m} \quad (47)$$

observer coefficient is as follows.

$$\beta_i = \alpha_i \omega_C^i \quad (48)$$

Here, α_i is binomial coefficient and ω_C is natural angular frequency of observer.

In Eq. (36), if $\tilde{\theta} = 0$, denote characteristic equation as follows.

$$\begin{aligned} D(s) &= |sI - (A_1 + \hat{\theta}A_2 - LC)| \\ &= s^{n+m} + \beta_1 s^{n+m-1} + (\beta_2 - 1)s^{n+m-2} + (\beta_3 - \beta_1)s^{n+m-2} + \\ &\quad \cdots + (\beta_{n+m-1} - \beta_{n+m-3})s + \beta_{n+m} \end{aligned} \quad (49)$$

Observer gain determined by Eq. (43) is satisfied with the following condition.

$$\beta_1 < \beta_2 < \cdots < \beta_i < \cdots < \beta_{n+m} \quad (50)$$

Thus, Eq. (49) has Hurwitz stability from Eq. (50).

Eventually, (n+m)-th order ADRC system with frequency estimation is stable.

Secondly, stability of ADRC using m order ESO with frequency estimation result in frequency estimation law of Eq. (42).

4. SIMULATION AND RESULTS

4.1. DISTURBANCE ERROR CHARACTERISTIC SIMULATION ACCORDING TO EXTENDED ORDER

For simulation analysis, the model of control plant is as follows from Eq. (1).

$$\left. \begin{aligned} \dot{x}_1 &= -ax_1 + Ku + d \\ y &= x_1 \end{aligned} \right\} \quad (51)$$

Here, $a = -\frac{1}{T}$, $K = \frac{k_I}{T}$

Disturbance d includes Eq. (2), lamp disturbance and parabolic disturbance.

$$d = M_0 + M(\omega) + M_V + M_a + M_t \quad (52)$$

Here

$$\begin{aligned} M_0 &= 0.001 \text{ Nm}, \quad M_\omega = -0.005 \text{sign}(\omega), \\ M_V &= 0.52t, \quad M_a = 0.698t^2, \\ M_t &= 0.39 \sin(1.33t) \end{aligned} \quad (53)$$

In Eq. (51), denote extended state as follows.

$$x_2 = -ax_1 + d$$

If extended order is $m = 3$ for rejecting the parabolic disturbance, m-th order extended state model is as follows.

$$\left. \begin{aligned} \dot{x}_1 &= Ku + x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= d^{(3)} \\ y &= x_1 \end{aligned} \right\} \quad (54)$$

ESO is as follows.

$$\left. \begin{aligned} \dot{\hat{x}}_1 &= Ku + \hat{x}_2 + \beta_1(y - \hat{y}) \\ \dot{\hat{x}}_2 &= \hat{x}_3 - \beta_2(y - \hat{y}) \\ \dot{\hat{x}}_3 &= \hat{x}_4 - \beta_3(y - \hat{y}) \\ \dot{\hat{x}}_4 &= \beta_4(y - \hat{y}) \\ \hat{y} &= \hat{x}_1 \end{aligned} \right\} \quad (55)$$

ADRC law and controller control law are designed by proposed design method as follows

$$\left. \begin{aligned} u &= -\frac{x_2}{K} + Ku_0 \\ u_0 &= k_1(r - y) + k_2\dot{r} \end{aligned} \right\} \quad (56)$$

Here, r is reference signal and k_1, k_2 are controller transfer coefficients, with $k_1 = 100, k_2 = 1$.

The value of β_i according to extended state order is used for $\omega_c = 50 \text{ 1/s}$.

$$\beta_1 = 4 \times 50, \beta_2 = 6 \times 50^2, \beta_3 = 4 \times 50^3, \beta_4 = 50^4 \quad (57)$$

In Eq. (52), test result for aperiodic disturbance is as follows

4.1.1. CHARACTERISTIC OF CONSTANT DISTURBANCE REJECTION ACCORDING TO m

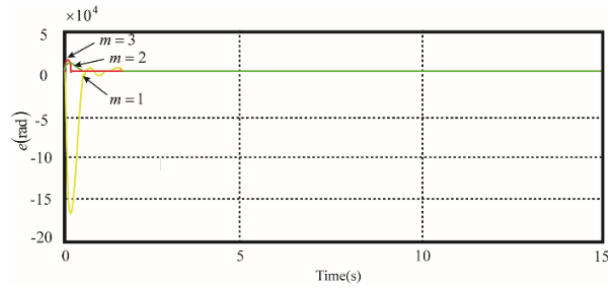


Figure 4: Characteristic of constant disturbance rejection according to m .

From Fig.4, we can see that error by disturbance is rejected to zero according to increasing m for constant disturbance.

4.1.2. CHARACTERISTIC OF CONSTANT SPEED DISTURBANCE REJECTION ACCORDING TO m

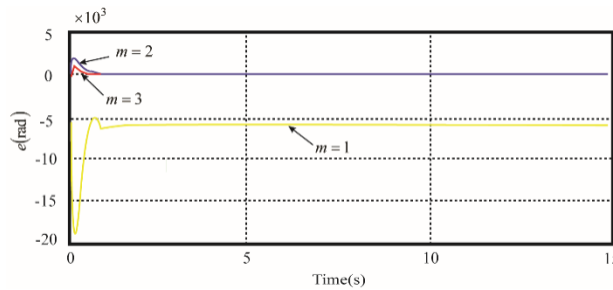


Figure 5: Characteristic of constant speed disturbance rejection according to m .

From Fig.5, in case of $m=1$, a certain size of error exists for lamp disturbance and in case of $m>1$, error by disturbance is rejected to be zero.

4.1.3. Characteristic of constant acceleration disturbance rejection according to m

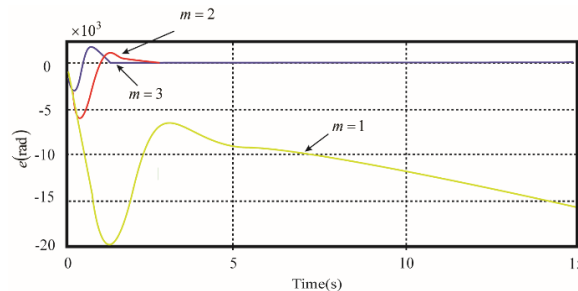


Figure 6: Characteristic of Constant acceleration disturbance rejection according to m .

From Fig.6, in case of $m=1$, the error by constant acceleration disturbance is increased according to time, in case of $m=2$, it has certain value and in case of $m=3$, it may be zero.

4.2. Harmonic disturbance error characteristic simulation according to extended order m

From Eq. (42), if $n=1$, $m=3$, $P=I$, frequency estimation law is as follows.

$$\begin{aligned}\dot{\hat{\theta}} &= -\lambda(X^T A_2^T P \tilde{X} + \tilde{X}^T P A_2 X) \\ &= -\lambda x_3 \tilde{x}_4 = -\lambda(\hat{x}_3 - \tilde{x}_3) \tilde{x}_4\end{aligned}\quad (58)$$

4.2.1. THE CASE OF SINGLE FREQUENCY HARMONIC DISTURBANCE

In simulation, disturbance is $d_2 = 0.39 \sin(1.33t)$.

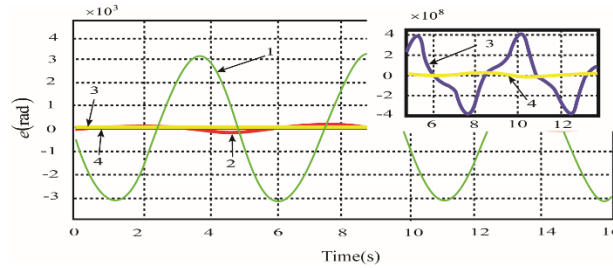


Figure 7: Error curves in case of disturbance- acting.

1- if $m=2$, error curve in case of not estimating frequency

2- if $m=3$, error curve in case of not estimating frequency

3- if $m=2$, error curve in case of estimating frequency

4- if $m=3$, error curve in case of estimating frequency

Fig.7 shows that in case of single frequency harmonic disturbance $0.39 \sin(1.33t)$ Nm, in case of not estimating frequency, and if $m=2$, maximum error amplitude is 3.1 mrad , if $m=3$, it is 0.1 mrad based on method of paper [16], and based on method of this paper using frequency estimation, if $m=2$, error is $4 \times 10^{-5} \text{ mrad}$, if $m=3$, the error is $2 \times 10^{-6} \text{ mrad}$.

Thus, disturbance rejection performance based on method of this paper is very high.

4.2.2. IN CASE OF VARIOUS FREQUENCY HARMONIC DISTURBANCE

In this simulation, disturbance is set as $d_2 = 0.39 \sin(1.33t) + 0.698 \sin(2t)$.

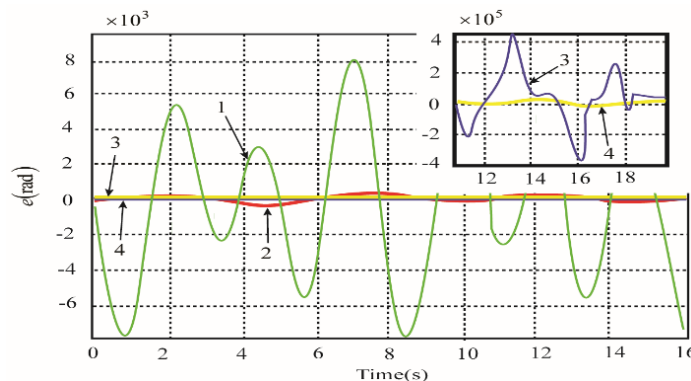


Figure 8: Error diagram in case of disturbance- if $m=2$, error curve in case of not estimating frequency

2- if $m=3$, error curve in case of not estimating frequency

3- if $m=2$, error curve in case of estimating frequency

4- if $m=3$, error curve in case of estimating frequency

Fig.8 shows that in case of single frequency harmonic disturbance $0.39\sin(1.33t) + 0.698\sin(2t)$, in case of not estimating frequency ,if $m=2$, maximum error amplitude is $8mrad$,if $m=3$, it is $0.29mrad$ based on method of paper [16], and based on method of this paper using frequency estimation, if $m=2$, error is $0.03mrad$, and if $m=3$, the error is $0.00mrad$.

Thus, disturbance rejection performance based on method of this paper is very high.

4.2.3. DISTURBANCE REJECTION PROPERTY TEST RESULT OF TARGET-INDICATING GIMBAL DEVICE

For rejecting disturbance of target-indicating gimbal device, the 2nd order ESO with frequency estimation is designed .

The result of characteristic test is as follows.

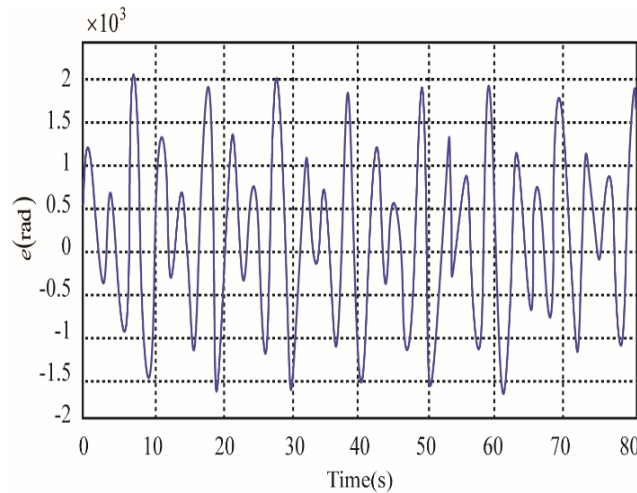


Figure 9: Error curve of target-indicating gimbal device combining with the 2nd order observer with fixed frequency estimation

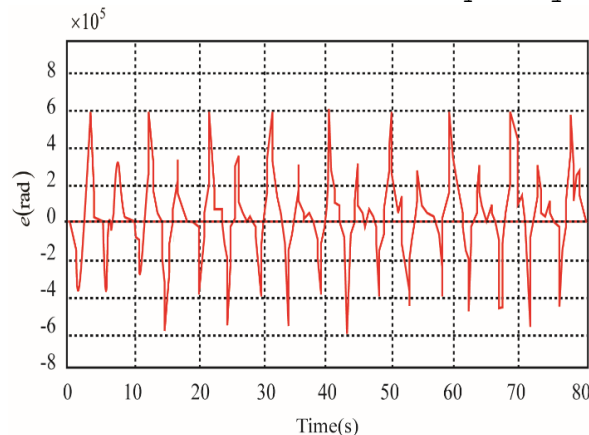


Figure 10: Error curve of ADRC system with the 2nd order observer with frequency estimation for target-indicating gimbal device.

In Table 3, as a result of disturbance rejection performance test of target-indicating gimbal device, in case of not estimating the frequency, the technical requirement is not satisfied.

Table 2: Disturbance Rejection Performance Test Result of Target-Indicating Gimbal Device

Test condition		First grade road	
No	Item	Reference value	Final value
1	ADRC with fixed frequency	$< 0.2 \text{ mrad}$	2.1 mrad
2	m^{th} order ADRC with frequency estimation	$< 0.2 \text{ mrad}$	0.063 mrad

But, in $(n + m)^{\text{th}}$ order ADRC with frequency estimation, the technical requirement is satisfied sufficiently.

CONCLUSIONS

This paper proposed the method to improve harmonic disturbance rejection performance of ADRC by combining m^{th} order ESO with frequency estimator, for satisfying high accuracy tracking performance of target-indicating gimbal device. Through proposed method, the following conclusions are obtained.

Firstly, for harmonic disturbance, in case of using m^{th} order ESO, constant, lamp disturbance and parabolic disturbance are rejected perfectly according to m.

Secondly, for single frequency harmonic disturbance, in case of using m^{th} order ESO with frequency estimation, harmonic disturbance is rejected perfectly according to m.

In addition, harmonic disturbance rejection performance improved greatly for various frequency harmonic disturbance.

As a result of disturbance rejection performance test of target-indicating gimbal device, in case of not estimating frequency, the technical requirement is not satisfied because of 2.1 mrad .

But in $(n + m)^{\text{th}}$ order ADRC with frequency estimation, the technical requirement is satisfied insufficiently because of 0.063 mrad .

It is confirmed that using ADRC having m^{th} order ESO with frequency estimation, it can improve disturbance rejection ability much more.

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